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Intermittency of temperature field in turbulent convection

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The scaling behavior of the temperature structure functions in turbulent convection is found to be different for length scales below and above the Bolgiano scale. Both sets of the exponents are well described by log-Poisson statistics. The parameter β_T which measures the degree of intermittency is the same for the two regimes of scales and is consistent with the corresponding value for the passive scalar field. A balance between thermal forcing and nonlinear velocity advection, which is a key ingredient leading to Bolgiano scaling, is also checked.

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A key issue in turbulence research is to make sense of the apparently random fluctuations of the various physical fields involved in a turbulent fluid flow. The seminal work of Kolmogorov in 1941 (K41) [1] predicts that the velocity structure functions have power-law scaling with the separating distance in the inertial range, the range of length scales that are smaller than those of energy input and larger than those affected directly by molecular dissipation, and the scaling exponents are proportional to the order of the structure function. This direct proportionality is equivalent to an independence of the functional form of the probability density function (PDF) of the velocity difference of the separating distance. Experimental observations confirm the power-law scaling but suggest that the exponent is a nonlinear function of the order. This deviation from the K41 results implies that the velocity field has scale-dependent statistical properties and is thus intermittent. Other physical quantities such as the temperature and pressure fields have also been found to be intermittent in turbulent flows.

Kolmogorov [2] and Obukhov [3] proposed a refined similarity hypothesis (RSH) which attributes intermittency of the velocity field to the spatial variations of the energy dissipation rate that were neglected in the K41 theory. Recently, She and Leveque [4] proposed a hierarchical relation for the moments of the locally averaged energy dissipation rate. With RSH, this leads to a similar relation for the velocity structure functions. These moment hierarchies were later shown to be naturally satisfied by log-Poisson statistics [5,6]. It has been found that the locally averaged energy dissipation rate [7], the velocity field [8,9], the passive scalar dissipation rate, [10] and the passive scalar field [11] in Navier-Stokes turbulence, and the velocity field in a class of shell models [12–14] all satisfy the log-Poisson moment-relations. It is thus interesting to investigate how general or universal log-Poisson intermittency is.

In this paper, intermittency of the temperature field in turbulent convection is studied. In convection, the dynamics is driven by the temperature difference, thus the temperature field is an active scalar. An interesting question is how turbulence is affected by buoyancy. One expects buoyancy to be important for length scales larger than the Bolgiano scale l_B which is defined as $\epsilon^{5/4}/[\chi^{3/4}(\alpha g)^{3/2}]$, where α is the volume expansion coefficient of the fluid, g the acceleration due to gravity, ϵ and χ are respectively the average energy and temperature dissipation rates. We shall see that the scaling behavior of the temperature structure functions is indeed different for length scales below and above l_B. Moreover, it is found that both sets of the exponents can be well described by log-Poisson statistics.

In the original works of Bolgiano [15] and Obukhov [16] for stably stratified turbulence (see [17] for a review), the temperature power spectrum is predicted to have a scaling $\sim k^{-7/5}$ when the wave number k satisfies $2\pi/k > l_B$. Results in agreement with the Bolgiano-Obukhov scaling have been reported for the temperature frequency power spectrum measured in helium [18] and water [19]. In the case of water, the frequency and wave number spectra were shown to coincide with each other in regions where a mean flow exists. A key ingredient leading to the Bolgiano-Obukhov scaling is the balance between thermal forcing and nonlinear velocity advection. Such a balance in turn relates the exponents of the velocity and the temperature structure functions. We shall check this relation using the exponents of temperature structure functions for length scales above l_B obtained in the present study and the exponents of velocity structure functions measured in a numerical study of Rayleigh-Bénard convection [20–22].

The temperature data analyzed in the present work were obtained in the well-documented Chicago experiment of low-temperature helium gas [23,24]. The experimental cell heated from below is cylindrical with a diameter of 20 cm and a height of $L = 40$ cm. The Rayleigh number (Ra) can be as high as 10^{15} and a mean circulating flow is found for Ra $\geqslant 10^8$. The temperature at the center of the cell, $T(t)$, was measured as a function of time t. We use Taylor hypothesis and evaluate the temperature difference between two times: $T_\tau(t) \equiv T(t+\tau) - T(t)$.

In an earlier work [25], the functional form of the PDF of T_τ was found to depend on τ which demonstrates that the temperature field is intermittent. From the τ-dependence, the dissipative and the circulation time scales, τ_d and τ_c, were also identified. The time scale τ_B corresponding to l_B is given by $\tau_B = \tau_c l_B / L$. It was shown [19] that l_B can be written as

$$l_B = \frac{\mathrm{Nu}^{1/2} L}{(\mathrm{Ra\,Pr})^{1/4}}, \tag{1}$$

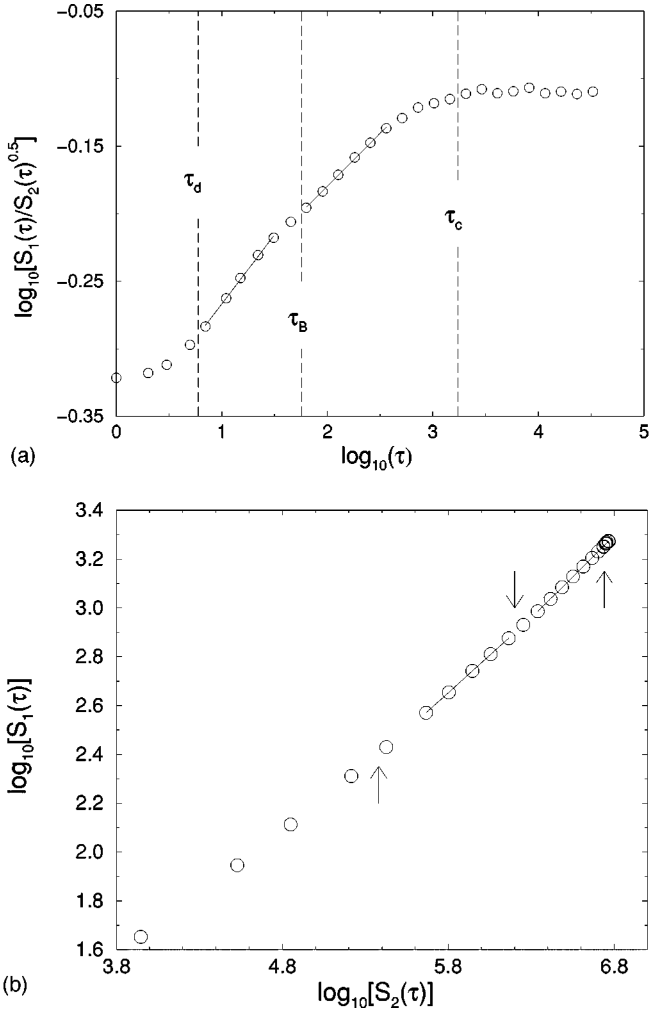

FIG. 1. (a) $\log_{10}[S_1(\tau)/S_2(\tau)^{1/2}]$ vs $\log_{10}(\tau)$ and (b) $\log_{10}[S_1(\tau)]$ vs $\log_{10}[S_2(\tau)]$ for $\mathrm{Ra}=7.3\times10^{10}$. The arrows, from left to right, indicate the data evaluated respectively at the three time scales: τ_d, τ_B, and τ_c. All times are in units of the sampling time $=1/320$ s. Two scaling regions are observed for $\tau<\tau_B$ and $\tau>\tau_B$.

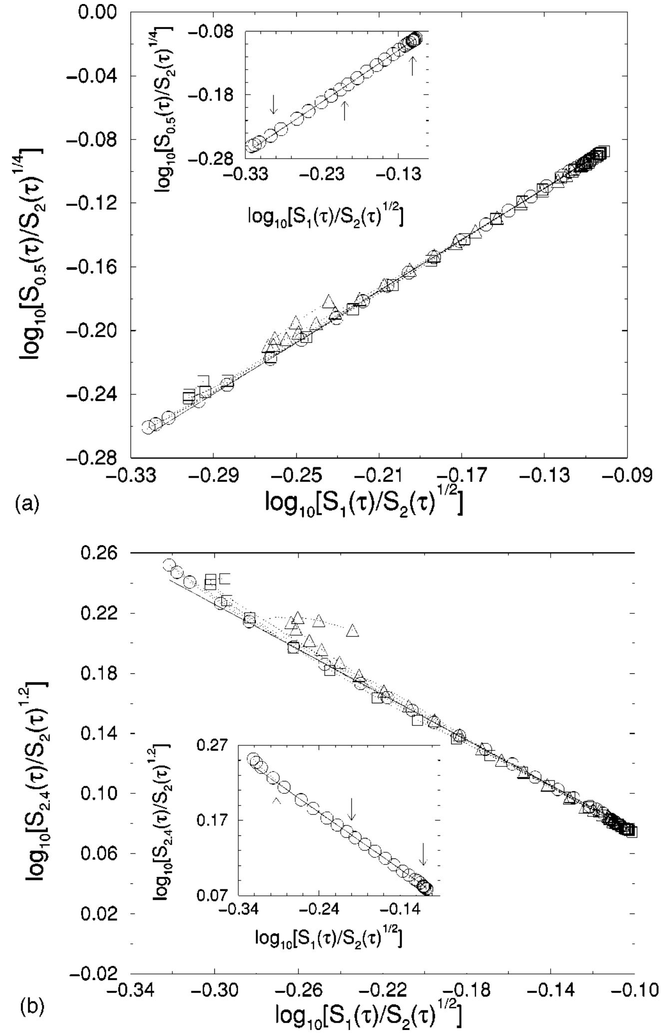

FIG. 2. $\log_{10}[S_p(\tau)/S_2(\tau)^{p/2}]$ vs $\log_{10}[S_1(\tau)/S_2(\tau)^{1/2}]$ with (a) $p=0.5$ and (b) $p=2.4$ for Ra ranges from 10^8 to 10^{15} (dashed lines). The scaling range first increases when Ra increases from 6.0×10^8 (squares) to 7.3×10^{10} (circles), then decreases as Ra is further increased to 5.8×10^{14} (triangles). The solid line is the best linear fit of the scaling range for $\mathrm{Ra}=7.3\times10^{10}$. In the insets, the data for $\mathrm{Ra}=7.3\times10^{10}$ (circles) and the best linear fit (solid line) to the scaling range are shown separately. The arrows, from left to right, indicate the data evaluated respectively at τ_d, τ_B, and τ_c.

where the Nusselt number (Nu) is the heat flux normalized by that when there was only conduction and the Prandtl number (Pr) is the ratio between kinematic viscosity and thermal diffusivity. With Eq. (1), τ_B is easily evaluated using the measured values of Nu, Ra, and Pr.

The temperature structure functions $S_p(\tau)\equiv\langle|T_\tau|^p\rangle$ are studied. For all the datasets with Ra ranges from 10^8 to 10^{15}, no discernible scaling is observed when $S_p(\tau)$ is plotted directly against τ in a log-log plot. On the other hand, scaling is observed when the normalized structure function, $\log_{10}[S_p(\tau)/S_2(\tau)^{p/2}]$, is plotted versus $\log_{10}\tau$. Interestingly, two scaling regions with different exponents, μ_p, are observed for τ below and above τ_B [see Fig. 1(a)]. Similarly, two regions with different exponents, α_p, are also observed when $\log_{10}S_p(\tau)$ is plotted against $\log_{10}S_2(\tau)$ [see Fig. 1(b)] using the idea of extended self-similarity (ESS) [26]. Thus, the temperature structure functions have different scaling behavior in the two regimes of length scales separated by the Bolgiano scale.

The scaling behavior is also studied using the idea of generalized extended self-similarity (GESS) [27]. In this case, $\log_{10}[S_p(\tau)/S_2(\tau)^{p/2}]$ is plotted as a function of $\log_{10}[S_1(\tau)/S_2(\tau)^{1/2}]$ and the slope is $h(p)=\mu_p/\mu_1$. As shown in Fig. 2, the scaling region revealed by these plots first increases as Ra increases from 10^8 to 10^{11} but then decreases for higher Ra. This observation is consistent with previous findings of changes in the behavior of the temperature frequency power spectrum [18] and the normalized temperature dissipation [28] but is not yet understood. For the present purpose, we concentrate on the data with the longest scaling range which were taken at $\mathrm{Ra}=7.3\times10^{10}$ (see insets of Fig. 2). The number of data points is 614 400. The three time scales are $\tau_d\approx6$, $\tau_B\approx60$, and $\tau_c\approx1750$ in units of the sampling time $=1/320$ s. Interestingly, one straight line is observed for $\tau_d<\tau<\tau_c$, which suggests that some common feature exists although the intermittency exponents μ_p and the relative scaling exponents α_p are different in the two regimes of scales (see below).

Let us first investigate whether the temperature structure functions obey the following log-Poisson hierarchy:

$$S^{(p+q,q)}(\tau)=A_{p,q}[S^{(p,q)}(\tau)]^{\beta_T^q}[S^{(\infty,q)}(\tau)]^{1-\beta_T^q}, \tag{2}$$

where $0<\beta_T<1$,

$$S^{(p,q)}(\tau)\equiv S_{p+q}(\tau)/S_p(\tau), \tag{3}$$

$$S^{(\infty,q)}(\tau)\equiv \lim_{p\to\infty} S^{(p,q)}(\tau), \tag{4}$$

and $A_{p,q}$ are constants independent of τ. Equation (2) generalizes the She-Leveque relation [4] to q not necessarily equal to 1. As a result, one can check the hierarchy without the need to calculate structure functions of very high order when small values of q are used. This is particularly useful when long datasets are not available. The log-Poisson hierarachy (2) predicts that

$$\mu_p=a\left[1-\beta_T^p-\frac{(1-\beta_T^2)}{2}p\right], \tag{5a}$$

$$\alpha_p=\frac{1-\beta_T^p+(b/a)p}{1-\beta_T^2+2(b/a)}, \tag{5b}$$

where $a\geqslant 0$ as required by the Hölder inequalities. The boundedness of the temperature field constrains b to be nonnegative if the scaling holds for $\tau\to 0$. Since $\tau_d/\tau_c\to 0$ as Ra $\to\infty$, $b\geqslant 0$ for $\tau<\tau_B$. On the other hand, as Nu scales like $\mathrm{Ra}^{1/2}$ for infinite Ra [29], τ_B/τ_c becomes independent of Ra, thus the value of b for $\tau>\tau_B$ is not constrained. The parameter β_T measures the degree of intermittency while a and b can be interpreted respectively as the codimension and the exponent of the most singular structures associated with the turbulent temperature field.

If Eq. (2) is valid, a plot of $\log_{10}[S^{(p+q,q)}(\tau_1)/S^{(p+q,q)}(\tau_2)]$ against $\log_{10}[S^{(p,q)}(\tau_1)/S^{(p,q)}(\tau_2)]$ by varying p for a given pair of τ_1, τ_2 and fixed q should give a straight line with slope β_T^q. The straight lines observed in Fig. 3 thus verify that Eq. (2) is satisfied. Note that the *same* slope is found for both $\tau_{1,2}$ below or above τ_B which shows that β_T is the same for the two regimes of scales. From the slopes of the straight lines, β_T is estimated to be 0.62 ± 0.02. Interestingly, this value is consistent with the corresponding value for the passive scalar field [11].

Equation (5a) implies that

$$h(p)=\frac{2(1-\beta_T^p)-(1-\beta_T^2)p}{(1-\beta_T)^2}. \tag{6}$$

Therefore, β_T can also be estimated using $h(p)$. As can be seen in Fig. 4, $h(p)$ is well fitted by Eq. (6) with $\beta_T=0.63$, which is consistent with the above estimate. With β_T found, a and b can be estimated from μ_p and α_p using Eqs. (5a) and (5b). The results are

$$a=\begin{cases}1.55\pm 0.1, & \tau<\tau_B\\ 1.2\pm 0.1, & \tau>\tau_B,\end{cases} \tag{7}$$

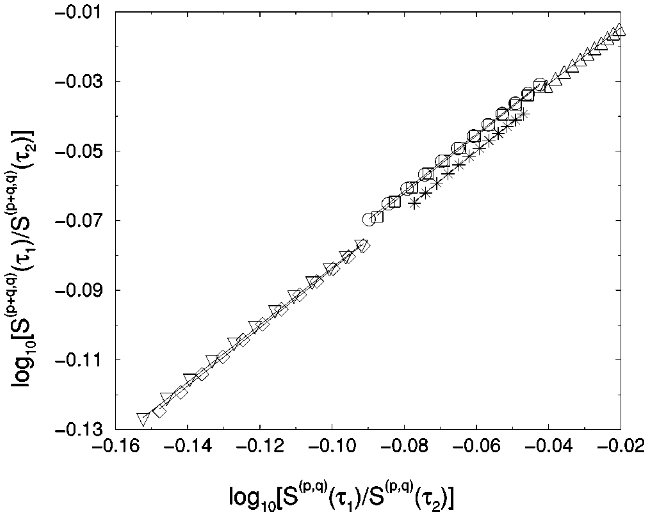

FIG. 3. $\log_{10}[S^{(p+q,q)}(\tau_1)/S^{(p+q,q)}(\tau_2)]$ plotted as a function of $\log_{10}[S^{(p,q)}(\tau_1)/S^{(p,q)}(\tau_2)]$ with $q=0.4$ for various pairs of τ_1 and τ_2. $\tau_1=8$, $\tau_2=32$ (diamonds); $\tau_1=10$, $\tau_2=20$ (stars); $\tau_1=10$, $\tau_2=50$ (inverted triangles); $\tau_1=64$, $\tau_2=256$ (squares); $\tau_1=100$, $\tau_2=500$ (circles); and $\tau_1=150$, $\tau_2=300$ (triangles). All times are in units of the sampling time $=1/320$ s. The solid lines are linear fits of the data.

$$b=\begin{cases}0, & \tau<\tau_B\\ -0.12\pm 0.01, & \tau>\tau_B.\end{cases} \tag{8}$$

If $S_p(\tau)\sim\tau^{\zeta_p}$, then $\zeta_p=a(1-\beta_T^p)+bp$. Also, we have $\mu_p=\zeta_p-p\zeta_2/2$ and $\alpha_p=\zeta_p/\zeta_2$. Therefore, one should get a straight line of slope ζ_2 when plotting μ_p versus $\alpha_p-p/2$. Straight lines are indeed observed in such plots and ζ_2 is estimated to be 0.93 ± 0.01 and 0.48 ± 0.005 for $\tau<\tau_B$ and $\tau>\tau_B$, respectively. These estimates are found to be consistent with the values evaluated using a, b and β_T. Note that ζ_2 is predicted to be 2/3 for a passive scalar in Navier-Stokes turbulence and 2/5 for Bolgiano scaling when intermittency is not taken into account.

An exact balance between thermal forcing and nonlinear velocity advection is denoted by

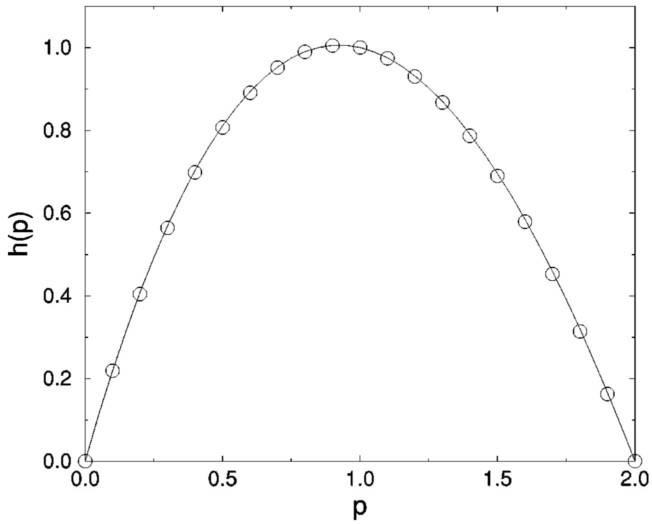

FIG. 4. Graph of the slopes $h(p)$ in GESS plots. The solid line is a fit using Eq. (6). The fitted value of β_T is 0.63.

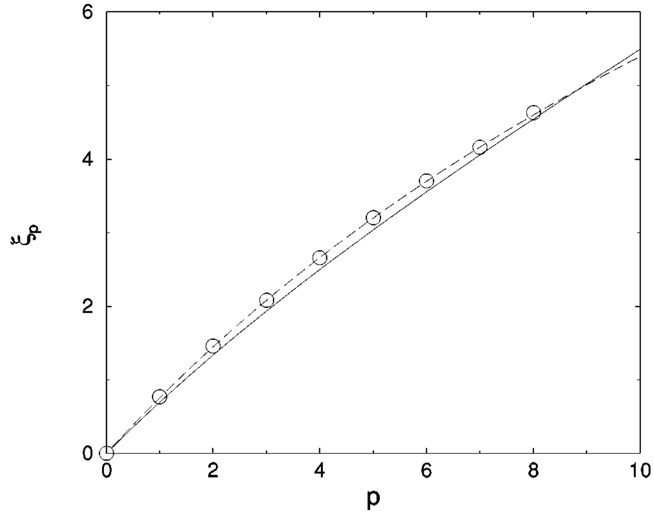

FIG. 5. Comparison of Eq. (10) (solid line) with the scaling exponents ξ_p obtained in a numerical study [22] (circles). The dashed line is $\xi_3\, \xi_p^{SL}$.

$$\frac{v_r^2}{r} \sim \alpha g T_r , \tag{9}$$

where v_r and T_r are the velocity and temperature differences across a separating distance r. This balance thus predicts that $\xi_p = \zeta_{p/2} + p/2$, where ξ_p is defined by $\langle |v_r|^p \rangle \sim r^{\xi_p}$. As a first attempt to directly check this balance, we estimate ζ_p by $\mu_p + p\zeta_2/2$ or $\zeta_2\, \alpha_p$. Our present results thus imply

$$\xi_p = a_{\tau>\tau_B}(1-\beta_T^{p/2}) + (1+b_{\tau>\tau_B})\frac{p}{2}. \tag{10}$$

As velocity time-data are difficult to measure in turbulent convection, we turn to numerical velocity measurements and compare Eq. (10) with the numerical values of ξ_p reported in Ref. [22]. In Fig. 5, we see that agreement is not found. In the same numerical study, it was further found that the relative exponents ξ_p/ξ_3 are consistent with those of Navier-Stokes turbulence [22] which are well described by the She-Leveque formula [4]: $\xi_p^{SL} = p/9 + 2[1-(2/3)^{p/3}]$ (see Fig. 5). This finding is also seen to be incompatible with Eq. (10) and $\beta_T = 0.62$.

In summary, intermittency of the temperature field in turbulent convection has been studied. As expected, buoyancy affects the statistics of turbulence for length scales larger than the Bolgiano scale, and two different sets of intermittency and relative scaling exponents are found for the two regimes of scales. Both sets of the exponents are found to be well described by log-Poisson statistics. The parameter β_T, which measures the degree of intermittency of the temperature field, is the same for the two regimes of scales, and is consistent with the corresponding value for the passive scalar field. On the other hand, a and b, which can be interpreted as the codimension and the exponent of the most singular structures, are different for scales below and above the Bolgiano scale. A simple balance between thermal forcing and nonlinear velocity advection as represented by Eq. (9), which is a key ingredient leading to the Bolgiano-Obukhov scaling, is also checked. Preliminary evidence suggests that this balance does not hold. Further analyses using simultaneous velocity and temperature data will be performed and results presented elsewhere.

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